Optimum Power Control for CDMA

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Abstract — **Power control laws to combat frequency-flat fading in CDMA are derived to maximize spectral efficiency for given $\frac{E_b}{N_0}$. Users are subject to independent fading with common distribution. Each transmitter adjusts its instantaneous transmitter power as a function of the fade level it experiences (without knowledge of other transmitters' fading conditions). We study the wideband limit of large number of users with randomly spread direct-sequence spread spectrum and four different receivers: the single-user matched filter that ignores interference, the decorrelator, the MMSE linear receiver and the optimum receiver.**

From the standpoint of Shannon capacity, the strategy of power equalization (or equal power control) used in CDMA commercial cellular systems is notoriously poor in terms of power efficiency, even worse than no power control. The various laws obtained in this paper retain the structural property of single-user optimum power control: no power is sent when the channel attenuation exceeds a certain threshold. The *water-filling* law [1] assigns the following instantaneous power when the channel power is θ:

$$p(\theta) = \left[\frac{1}{\lambda} - \frac{1}{\theta}\right]^+ \tag{1}$$

with $[a]^+ = \max\{0, a\}$ and λ chosen so as to satisfy the average power constraint.

We have shown that even if the transmitter has only partial knowledge of the fade level, unbounded slow fading allows reliable transmission at arbitrarily low $\frac{E_b}{N_0}$.

The single-user results do not offer much encouragement for the prospects of power control above the very low SNR region. In contrast, in CDMA optimum power control offers performance gains even at high $\frac{E_b}{N_0}$. The nature of those gains turns out to depend heavily on the receiver.

The optimum power-control law for the **matched filter** receiver has the structure

$$p(\theta) = \left[\frac{1}{\lambda_1 + \lambda_2\theta} - \frac{1+\zeta\beta}{\theta}\right]^+ \tag{2}$$

For any number of users per chip, β, at high $\frac{E_b}{N_0}$ truncated power equalization is asymptotically optimum and it wipes out the fading penalty. Thus, the (interference-limited) limiting value of capacity for $\frac{E_b}{N_0} \to \infty$ is raised by eliminating the Jensen penalty.

The optimum power control law for the **decorrelator** is of the water-filling type

$$p(\theta) = \left[\frac{1}{\lambda} - \frac{1}{(1-\beta)\theta}\right]^+. \tag{3}$$

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As in the single-user channel, the effect of optimum power control for the decorrelator is to extend the minimum required $\frac{E_b}{N_0}$ to $-\infty$ and to improve the low-$\frac{E_b}{N_0}$ spectral efficiency. This effect is more pronounced than in the single-user channel because of the increased minimum $\frac{E_b}{N_0}$ of the decorrelator. However, power control is not a sensible design choice here. As for single-user channels, at high $\frac{E_b}{N_0}$, it hardly offers any improvement. At low $\frac{E_b}{N_0}$ the matched filter with power control offers better performance and lower complexity.

For the **MMSE receiver**, the optimum power control law has the structure

$$p(\theta) = \frac{1}{\lambda_1}\left[1 + \sqrt{1 - \frac{\lambda_1\lambda_2}{\theta\eta}} - \frac{\lambda_1}{\theta\eta}\right]^+ \tag{4}$$

Power control helps the MMSE receiver tremendously particularly if $\beta > 1$ and $\frac{E_b}{N_0}$ is high. Whereas without power control the spectral efficiency is bounded if $\beta > 1$, optimum power control (or truncated power equalization) has a "population control" effect that makes the receiver operate with an effective number of interferers that is lower than the spreading gain, a scenario in which the MMSE spectral efficiency is unbounded with $\frac{E_b}{N_0}$.

Curiously, the structure of the optimum power control law for the **optimum multiuser receiver** is simpler than those of the linear receivers:

$$p(\theta) = (1 + \beta\bar{g}\mathsf{SNR})\left[\frac{1}{\bar{g}} - \frac{1}{\theta}\right]^+ \tag{5}$$

The power control law (5) is obtained using a formula for the nonlinear gain found in [2].

With sufficiently large β the potential gains achievable by optimum power control and an optimum receiver are staggering. It is possible to outperform even the single-user channel without fading. The strategy for large β is to allow transmissions by only those users whose channel gains are exceedingly favorable (no more than $\log K$ in Rayleigh fading), but with very small per-symbol SNR. With this scheme and any unbounded fading distribution we obtain the following result: *It is possible to achieve any desired spectral efficiency b/s/Hz at any $\frac{E_b}{N_0}$ provided β is allowed to be large enough.* The increase of capacity with β is extremely slow, however. Overly ambitious b/s/Hz-$\frac{E_b}{N_0}$ pairs come, as we would expect, at a huge complexity cost.

Full details on the results and proofs of this paper can be found in [2].

References

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